Centre Number						Candidate Number				
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Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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6	
7	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2015

Mathematics MM03

Unit Mechanics 3

Wednesday 3 June 2015 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 A formula for calculating the lift force acting on the wings of an aircraft moving through the air is of the form

$$F = k\,v^{\alpha} A^{\beta} \rho^{\gamma}$$

where F is the lift force in newtons,
k is a dimensionless constant,
v is the air velocity in $\mathrm{m\,s^{-1}}$,
A is the surface area of the aircraft's wings in $\mathrm{m^2}$, and
ρ is the density of the air in $\mathrm{kg\,m^{-3}}$.

By using dimensional analysis, find the values of the constants α, β and γ.

[6 marks]

QUESTION PART REFERENCE

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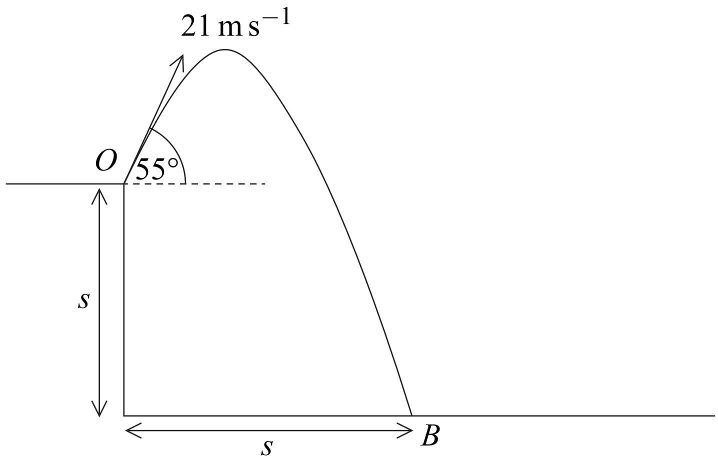
2 A projectile is launched from a point O on top of a cliff with initial velocity $u\,\mathrm{m\,s^{-1}}$ at an angle of elevation α and moves in a vertical plane. During the motion, the position vector of the projectile relative to the point O is $(x\mathbf{i} + y\mathbf{j})$ metres where $\mathbf{i}$ and $\mathbf{j}$ are horizontal and vertical unit vectors respectively.

(a) Show that, during the motion, the equation of the trajectory of the projectile is given by

$$y = x\tan\alpha - \frac{4.9x^2}{u^2\cos^2\alpha}$$

[5 marks]

(b) When $u = 21$ and $\alpha = 55°$, the projectile hits a small buoy B. The buoy is at a distance s metres vertically below O and at a distance s metres horizontally from O, as shown in the diagram.



(i) Find the value of s.

[3 marks]

(ii) Find the acute angle between the velocity of the projectile and the horizontal just before the projectile hits B, giving your answer to the nearest degree.

[5 marks]

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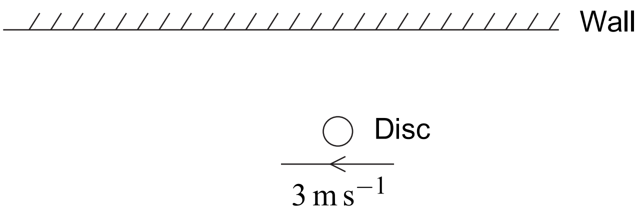
3 A disc of mass $0.5\,\mathrm{kg}$ is moving with speed $3\,\mathrm{m\,s^{-1}}$ on a smooth horizontal surface when it receives a horizontal impulse in a direction **perpendicular to its direction of motion**. Immediately after the impulse, the disc has speed $5\,\mathrm{m\,s^{-1}}$.

(a) Find the magnitude of the impulse received by the disc.

[3 marks]

(b) Before the impulse, the disc is moving parallel to a smooth vertical wall, as shown in the diagram.



After the impulse, the disc hits the wall and rebounds with speed $3\sqrt{2}\,\mathrm{m\,s^{-1}}$.

Find the coefficient of restitution between the disc and the wall.

[4 marks]

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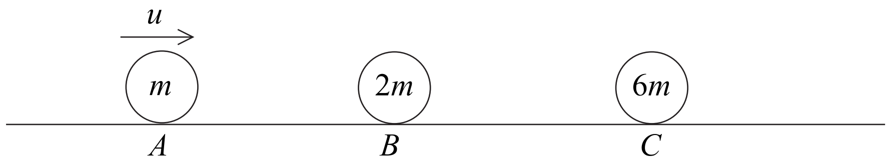
4 Three uniform smooth spheres, A, B and C, have equal radii and masses m, $2m$ and $6m$ respectively. The spheres lie at rest in a straight line on a smooth horizontal surface with B between A and C. The sphere A is projected with speed u directly towards B and collides with it.



The coefficient of restitution between A and B is $\frac{2}{3}$.

(a) (i) Show that the speed of B immediately after the collision is $\frac{5}{9}u$.

(ii) Find, in terms of u, the speed of A immediately after the collision.

[6 marks]

(b) Subsequently, B collides with C. The coefficient of restitution between B and C is e.

Show that B will collide with A again if $e > k$, where k is a constant to be determined.

[8 marks]

(c) Explain why it is not necessary to model the spheres as particles in this question.

[2 marks]

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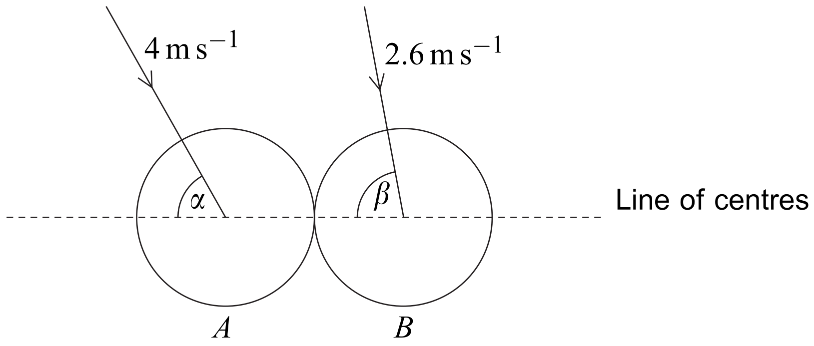
5 Two smooth spheres, A and B, have equal radii and masses $2\,\text{kg}$ and $1\,\text{kg}$ respectively. The spheres move on a smooth horizontal surface and collide. As they collide, A has velocity $4\,\text{m s}^{-1}$ in a direction inclined at an angle α to the line of centres of the spheres, and B has velocity $2.6\,\text{m s}^{-1}$ in a direction inclined at an angle β to the line of centres, as shown in the diagram.



The coefficient of restitution between A and B is $\frac{4}{7}$.

Given that $\sin\alpha = \frac{4}{5}$ and $\sin\beta = \frac{12}{13}$, find the speeds of A and B immediately after the collision.

[11 marks]

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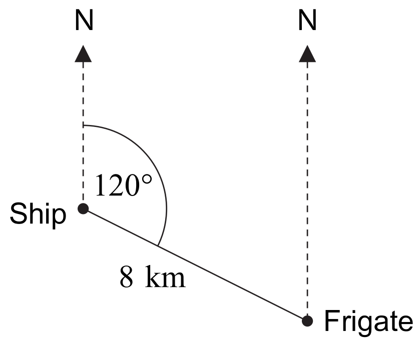
6 A ship and a navy frigate are a distance of $8\,\text{km}$ apart, with the frigate on a bearing of $120°$ from the ship, as shown in the diagram.



The ship travels due east at a constant speed of $50\,\text{km}\,\text{h}^{-1}$. The frigate travels at a constant speed of $35\,\text{km}\,\text{h}^{-1}$.

(a) (i) Find the bearings, to the nearest degree, of the two possible directions in which the frigate can travel to intercept the ship.

[5 marks]

(ii) Hence find the **shorter** of the two possible times for the frigate to intercept the ship.

[5 marks]

(b) The captain of the frigate would like the frigate to travel at less than $35\,\text{km}\,\text{h}^{-1}$.

Find the minimum speed at which the frigate can travel to intercept the ship.

[3 marks]

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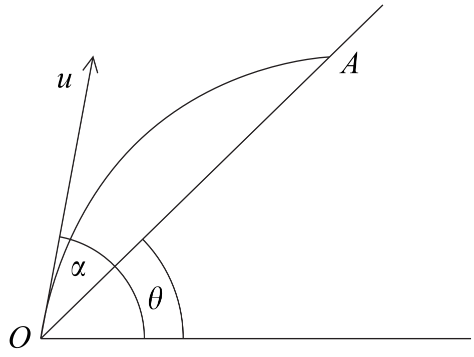
7 A particle is projected from a point O on a plane which is inclined at an angle θ to the horizontal. The particle is projected up the plane with velocity u at an angle α **above the horizontal**. The particle strikes the plane for the first time at a point A. The motion of the particle is in a vertical plane which contains the line OA.



(a) Find, in terms of u, θ, α and g, the time taken by the particle to travel from O to A.

[4 marks]

(b) The particle is moving horizontally when it strikes the plane at A.

By using the identity $\sin(P-Q) = \sin P \cos Q - \cos P \sin Q$, or otherwise, show that

$$\tan\alpha = k\tan\theta$$

where k is a constant to be determined.

[5 marks]

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END OF QUESTIONS

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